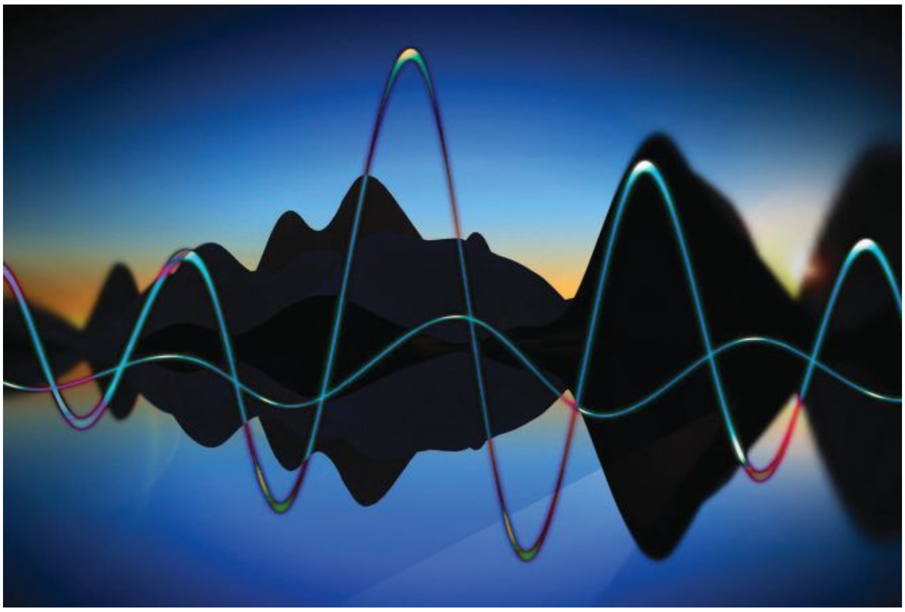
Tech Topic

SONIC ATMOSPHERE

Calculating the speed of sound in air.

by Merlijn van Veen



In this article we'll investigate how the speed of sound in air is, for all intents and purposes, exclusively temperature dependent within the audible bandwidth of our typical applications. There are some popular misconceptions on this subject related to pressure, density, and other effects that are addressed here.

The speed of sound is the distance traveled per second through an elastic medium. The medium is composed of molecules held together by intermolecular forces. Sound energy passes through the medium by compressing and expanding these bonds.

PRESSURE & DENSITY EFFECTS

The speed of sound through a medium depends predominantly on its stiffness and density. Stiffness is a metric that describes the resistance of a substance to uniform compression and is measured in Pascal, which equals 1 Newton per square meter (approximately 0.1 kilograms per square meter or 0.02 pounds per square foot). Density is the ratio of weight to volume measured in kilograms per cubic meter.

In general, the speed of sound through any medium (solid, liquid or gaseous) is described by Equation 1. I've singled out the relevant parameters to keep it simple.

Equation 1

$$c = \sqrt{\frac{K}{\rho}}$$

Where:

- c is the speed of sound measured in meters per second
- K is the elasticity modulus measured in Pascal
- ρ (Greek letter rho) is density measured in kilograms per cubic meter

Notice how increasing elasticity (stiffness) accelerates sound speed (higher numbers mean less elastic and more stiff) and rising density decelerates.

Consider the fact that sound travels faster through water than air. At room temperature water is over 800 times more dense than air. Equation 1 seems to indicate that the high density should decrease the medium's sound speed, which is quite counterintuitive. And yet water is over 15,000 times less elastic (stiffer) than air, offsetting the comparatively small density increase, making the water sound speed more than four times faster than through air.

We could therefore expect that density will play a significant part in the sound speed of air. The fact that pressure and density change with altitude leads to the common misconception that we'd need to set delays differently on Mt. Everest than the Riviera. However, for air as a medium there's a twist.

The air we use for acoustic transmission acts like an ideal gas (conveniently simplifying our equations). The pressure/density relationship for an ideal gas is shown in Equation 2, which holds as long as the medium's thermodynamic properties, molecular composition and volume are unchanged. Again I've singled out the relevant parameters in order to keep it simple.

Equation 2

$$\frac{p}{\rho} = T$$

Where:

- p is pressure measured in Pascal
- ρ (Greek letter rho) is density measured in kilograms per cubic meter
- T is temperature measured in Kelvin

The Kelvin scale is a shifted Celsius scale where 0 Kelvin, the lowest possible temperature, equals -273.15 degrees Celsius (-459.67 degrees Fahrenheit). For gaseous media there's a push-pull relationship between pressure and density. Equation 2 shows that pressure and density are proportional to each other when temperature is matched. The ratio of pressure over density is constant, bound by temperature. They follow each other in lockstep.

The elasticity modulus for air, mentioned earlier in Equation 1, is proportional to pressure. As pressure rises, the medium becomes less elastic (stiffer), accelerating the sound speed. However, as pressure goes up, density must follow suit (Equation 2), thereby decelerating proportionally to create a sound speed stalemate (with constant air temperature). Therefore, the speed of sound in Johannesburg at an altitude of 1,750 meters (5,800 feet) is identical to Amsterdam at sea level (as long as the temperature is matched).

TEMPERATURE EFFECTS

Let's now isolate the temperature effects while maintaining a constant amount of substance and volume that constitute density.

Temperature rise accelerates the molecules, increases the pressure, which makes the medium less elastic (stiffer) and therefore faster. Equation 3 provides accurate sound speed calculation for dry air based on temperature, taking into account the medium's thermodynamic properties.

Equation 3

$$c(m/s) = 331.3\sqrt{1+\frac{T(^\circ C)}{273.15}}$$

Where:

- c is the speed of sound measured in meters per second
- T is temperature measured in degrees Celsius

For habitable conditions where we actually get to enjoy sound without turning into popsicles or burning up, a simpler approximation (Equation 4) will suffice.

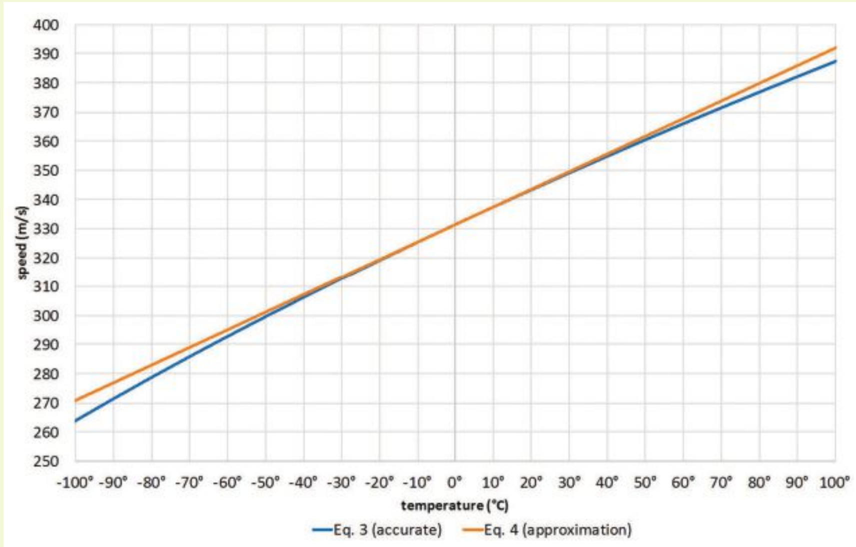


Figure 1 – The speed of sound in dry air.

Equation 4

$$c(m/s) = 331.3 + [0.606 \cdot T(^\circ C)]$$

Where:

- c is the speed of sound measured in meters per second
- T is temperature measured in degrees Celsius

This simplified equation is close enough for live sound applications and accurate within 0.5 percent, as evident from its close resemblance to the long form equation (**Figure 1**).

HUMIDITY EFFECTS

We previously showed how pressure and density effects offset each other as long as we kept temperature and molecular/thermodynamic properties constant. Now let's explore molecular/thermodynamic changes via the mechanism known as "humidity," which describes how the air medium varies from dry to wet. We previously compared the mountaintop to the beach. Now it will be the desert versus the rain forest.

Our typical transmission medium contains both air (a combination of oxygen, nitrogen and other molecules) and water molecules (good old H2O). The term humidity describes the ratio of water vapor and air. "High" humidity air has a greater proportion of water vapor mixed into the medium than "low" humidity air. Therefore, a humidity change is, de facto, a change in the medium's molecular composition.

The compound pressure of both water vapor and dry air make the medium less elastic (stiffer), resulting in acceleration. Humid air is less dense than dry air because water's molar mass (the density on an atomic level) is less than that of dry air, which also accelerates sound speed.

However, water's thermodynamic properties are slightly different from the gases oxygen and nitrogen. Therefore, a humidity change is more than just a pressure and density change. It is also a revision of the medium's transmission properties, resulting in a deceleration of the speed of sound.

Overall it suffices to say that wet can be slightly faster. Aha! So wet air is indeed faster than dry air! But as we shall see, the change is so negligible it will never factor into our sound system performance.

Figure 2 (following the research of Owen Cramer*) shows every possible outcome for combinations of temperature and relative humidity. Each color division represents a sound speed change of 1 m/sec (approximately 0.3 percent). Imagine 0 percent as the driest desert and 100 percent RH as the point where air is saturated and cloud forming can occur.

The chart's left side (blue area showing cold and slow air) shows that even a transition from dry ice to wet ice (bottom to top) does not change the sound speed by even a single meter/second. This is because cold air can hold less water vapor until saturated than hot air. The right side (red area for hot and fast air) shows that moving from the Sahara to the Amazon would have a much larger effect, but still so small that it would be the least of our problems even though hot air can contain more water vapor until saturated.

It takes a relative humidity change of 100 percent to cause a negligible 1 meter per second increase in sound speed. Any changes in sound transmission quality will be due much more to the umbrellas than the transmission speed.

To put this in perspective, the most extreme humidity change possible would have the equivalent effect of a mildly noticeable temperature change (a mere 1.7 degrees C). It's clear that we don't need an altimeter or a humidistat to set delays or understand the speed of sound.

For all but the most extreme situations imaginable, we can solve sound speed with a thermometer and calculator alone. We can stick to Equations 4 through 6 with errors of less 0.3 percent, i.e., 1 meter (3 feet) per second.

Equation 5

$$c(ft/s) = 1{,}087 + [2 \cdot T(^\circ C)]$$

Where:

- c is the speed of sound measured in feet per second
- T is temperature measured in degrees Celsius

Equation 6

$$c(ft/s) = 1{,}052 + [10/9 \cdot T(^\circ F)]$$

Where:

- c is the speed of sound measured in feet per second
- T is temperature measured in degrees Fahrenheit

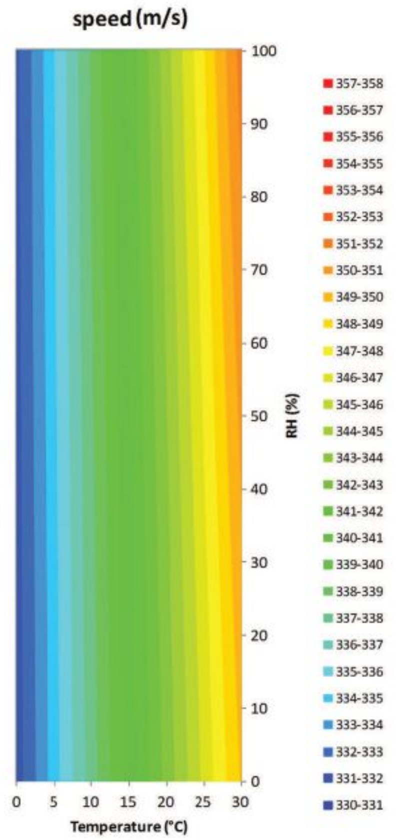


Figure 2 – The speed of sound in humid air.

[*] Owen Cramer, "The variation of the specific heat ratio and the speed of sound in air with temperature, pressure, humidity, and CO2 concentration," *Journal of the Acoustical Society of America*, 1993 (J. Acoust. Soc. Am. 93, 2510), http://dx.doi.org/10.1121/1.405827 LSI

Based in The Netherlands, ***Merlijn van Veen*** *(https://www.merlijnvanveen.nl) is a consultant specializing in sound system design and optimization, and he's also a noted audio educator. Merlijn offers special thanks to Bob McCarthy for his help with this article and "increasing the wits by 10 dB."*